

Experimental realization of the Furuta pendulum to emulate the Segway motion

G.Pujol¹, L. Acho¹, A. Nápoles², and V. Pérez-García³

¹Department of Applied Mathematics III, Universitat Politècnica de Catalunya-BarcelonaTech (EUETIB), Comte Urgell 187, 08036 Barcelona, Spain. (email: gisela.pujol@upc.edu, leonardo.acho@upc.edu).

²Department of Mechanical Engineering, Universitat Politècnica de Catalunya-BarcelonaTech (EUETIB), Comte Urgell 187, 08036 Barcelona, Spain. (email: amelia.napoles@upc.edu).

³Department of Strength of Materials and Structural Engineering, Universitat Politècnica de Catalunya-BarcelonaTech (EUETIB), Comte Urgell 187, 08036 Barcelona, Spain. (email: vega.perez@upc.edu).

Abstract

A Laboratory device is designed to emulate the Segway motion, modifying the Furuta pendulum experiment. To copy the person on the Segway transportation unit to the Furuta pendulum, a second pendulum is added to the main one. Using LMI theory, the control objective consists to manipulate the base of the Furuta pendulum according to the inclination of the added pendulum whereas the coupled pendulums are maintained near to theirs upright positions. According to the inclination of the second pendulum, the base will rotate as long as the second pendulum remains inclined, emulating the Segway behavior. The base rotation will depend on the side of the inclination too. Experimental results are offered too (video link: <http://youtu.be/SHQdW3k3qiE>).

Keywords: Robust control, LMI, Laboratory device.

1 Introduction

Under-actuated dynamical systems are those that have more degrees-of-freedom than control inputs. Examples include spacecrafts, underwater autonomous vehicles and mobile robots. Control and stabilization of these systems are challenging tasks and are currently hot topics of research for both engineers and applied mathematicians [1, 2]. New stabilization strategies are validated and tested on classical benchmark systems such as the ‘ball on a beam’ and ‘inverted pendulum’ systems [3, 4]. An interesting problem comes from introducing some kind of perturbation to these dynamical systems, to study the robustness in front external disturbances [5, 6]. Also, one such systems which has drawn the attention of control researchers is the Mobile Inverted Pendulum (MIP) which is a two-wheeled robot with a central body that carries a payload. The robot has the advantage of having a small footprint in addition to its ability to turn about its central axis. A commercial variant of the MIP is the well known Segway [7]. We can define a Segway Robotic Mobile Platform (RMP) as self-balancing personal transport unit which is similar to the classic control inverted pendulum stabilization problem. In this paper, a laboratory device is designed to create a commanded disturbance to the Furuta pendulum (rotational inverted pendulum). This pendulum is modified by adding a second inverted pendulum coupled to the main one (see Figure 1). The induced motion on the second pendulum causes displacement of the center mass of the system, producing a kind of perturbation similar to that presented on MIP transportation unit. Roughly speaking, modifying the position of an inverted pendulum (the person driving the Segway), the displacement of the base (the Segway) is obtained. In practice, we modify an existent experiment Furuta pendulum presented in Figure 2 (provided by ECP-systems) by replacing weight m_w by this second inverted pendulum. Figures 3 and 4 present this new experimental realization. This

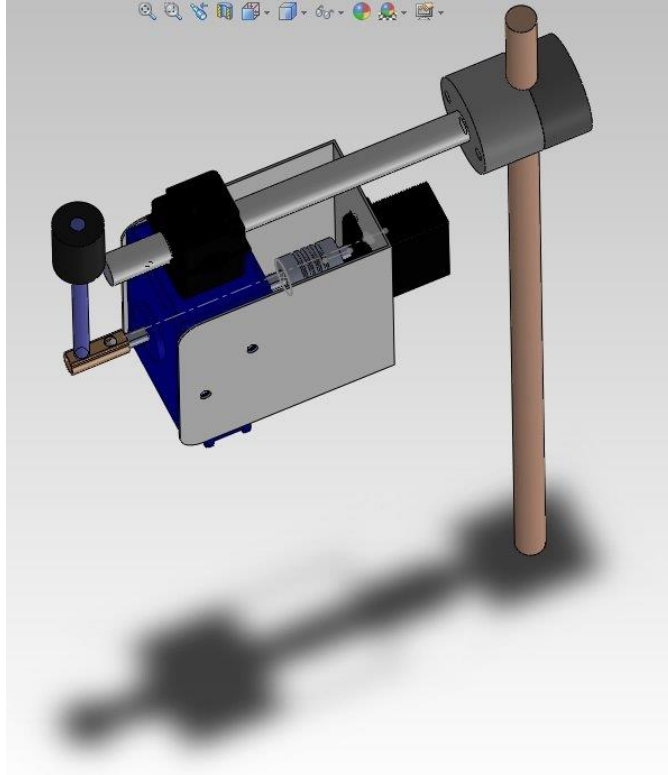


Figure 1: Laboratory device.

external perturbation (second pendulum) is quantized by measuring its angular position via an encoder from *Passport* company. So, we can study directly how this disturbance actuates on our inverted pendulum.

Our control objective is to experiment with this dynamic system in order to prove robust stability of an output feedback controller. Many nonlinear results exist in the bibliography, incorporating experimental applications [8, 9, 10, 11], but the purpose of the present paper is to design an easy control algorithm that solves a complex nonlinear problem. For linear systems, H_∞ control theory offers the possibility of including robustness considerations explicitly in the design and the opportunity to formulate physically meaningful performance ob-

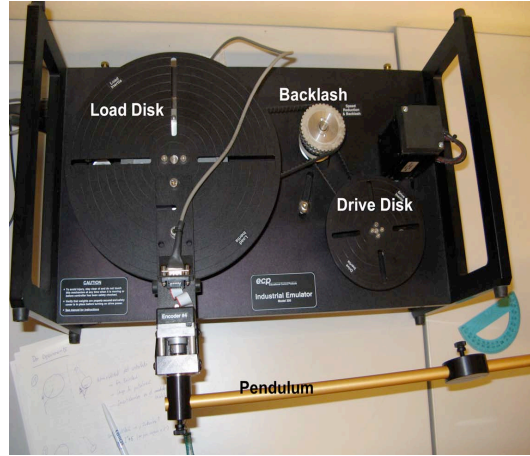


Figure 2: Basic pendulum assembly control (Model M220) of ECP systems company [18].

jectives that can be expressed as H_∞ design specifications [12, 13], and solved via the Linear Matrix Inequality (LMI) techniques [14, 15]. It is convenient to point that firstly the H_∞ linear controller clammed in [12] was implemented in our Furuta experiment, but experimentally the closed-loop systems was unstable [6]. Based on LMI techniques, [16] presents a hierarchy intelligent control scheme for a Segway vehicle, but without real experimentation. Also, a fuzzy control is designed and implemented to the same problem in [17]. With simplicity in mind, we propose a linear approach. An output feedback LMI controller is obtained from the unperturbed model (the one without the second inverted pendulum). The stability is proved, and the controller performance is evaluated experimentally. According to experiments, the output feedback LMI controller is robust against this kind of perturbation. In particular, its possible realization as a Segway navigation control unit center is proved. For control application on the Segway, we use a LMI- H_∞ controller originally designed to stabilize the inverted pendulum. We modify it to fit the navigation control objective. This change consists to do not take into account the displacement variable of the

load disk (see Figure 2), that is, the output control ignores the disk position. Doing this modification on the control law, the disk position moves freely and we obtain navigation.

This paper presents a laboratory device motivating first the Segway realization. Then, mathematical model as well as the control problem are firstly described in Section three. Experimental test and results are commented in Section four, showing controller robustness. Finally, Section five gives the conclusions.

2 Laboratory device design

2.1 Discussion: segway inspiration

As it was said in the introduction, under-actuated dynamical systems are those that have more degrees-of-freedom than control inputs. One such systems which has drawn the attention of control researchers is the MIP. The robot has the advantage of having a small footprint in addition to its ability to turn about its central axis. A commercial variant of the MIP is the well known Segway [7]. We can define a Segway Robotic Mobile Platform (RMP) as self-balancing personal transport unit which is similar to the classic control inverted pendulum stabilization problem. The navigation of this unit is due to, basically, the displacement of the center of mass of the body. This line of reasoning motivates us our design showed in Figures 3 and 4. With this new device, the position of the center of gravity of the second inverted pendulum is modified by hand, to induce a perturbation which produces displacement of the Furuta pendulum. At this respect, the main pendulum, attached to the rotating base (Figures 2, 3, 4 and 5), can emulate the Segway machine, with the difference that the Segway moves linearly, but the Furuta pendulum rotates. The second inverted pendulum can

emulate a person (Figure 5).

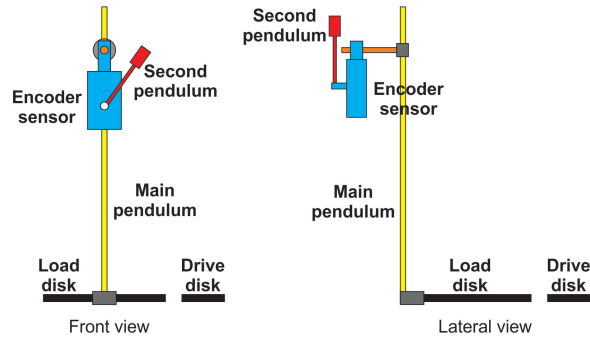


Figure 3: Coupled pendulums experiment: concept design (front and lateral view). On this picture, in blue, the encoder sensor. In yellow, the main pendulum. In red, the second pendulum coupled to the main one.

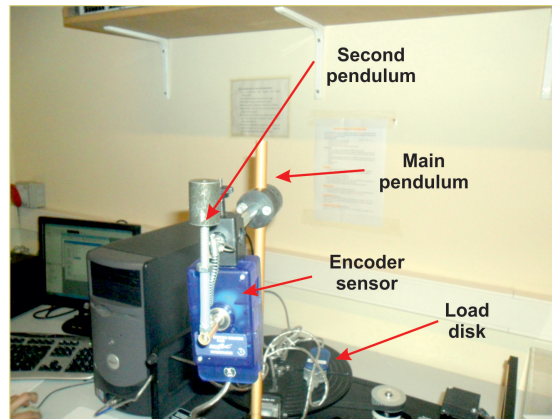


Figure 4: Coupled pendulums experiment: A new realization.

Figure 5: Basic Segway inspiration scheme.

2.2 Description of the laboratory device

The laboratory device is described in Figure 4. The original experiment is formed by the load and drive disk, with the main pendulum, as Figure 2 shows. These parts emulate the Segway. In this experiment, we consider the load and drive disk as an ensemble, and we define it as the load disk. At $y_m = 32$ cm (Figure 6), we place the new device laboratory that emulates the person driving the Segway. Its total weight is 48 gr. The second pendulum is added to change its center of mass. Its weight is 21 gr. It is possible to modify the weight and structure of the second pendulum changing its end-piece. We have angular position measurements of the disk (θ_1) and the main pendulum (θ_2). Also, the angular position of the second pendulum is measured off-line from the control-loop via an encoder.

3 Dynamical system equations and control statement

3.1 Dynamical system equations

The rotating base system driven by a motor shown in Figures 2 and 6 is representative of the pendulum attached to experimental Model 220 apparatus [18], where $\theta_1(t)$ is the magnitude of angular rotation of the disk with respect to O_1 and $\theta_2(t)$ is the magnitude of angular rotation of the pendulum with respect to O_2 . In the case study described in this paper, the mass set at P (weight m_w in Figure 6) is changed by a second inverted pendulum with total mass 48gr (Figures 3 and 4).

Figure 6: Diagram representation of Furuta pendulum with rotating base [18].

The equations of motion for the unperturbed case are obtained from Lagrange's equations and may be linearized in the case of $\theta_1 = 0$ and $\theta_2 = 0$ [18]:

$$\ddot{\theta}_1 = \frac{1}{p} \left(-\dot{\theta}_1 \bar{J}_z - m^2 l_{cg}^2 R_h g \theta_2 + \bar{J}_z u \right), \quad (1)$$

$$\ddot{\theta}_2 = \frac{1}{p} \left(m R_h l_{cg} \dot{\theta}_1 - m g l_{cg} (\bar{J}_1 + J_y) \theta_2 - m R_h l_{cg} u \right), \quad (2)$$

where $p = \bar{J}_z(\bar{J}_1 + J_y) - (m R_h l_{cg}^2)$ is normalized, $\bar{J}_1 = J_1 + m R_h^2$ and $\bar{J}_z = J_z + m l_{cg}$; J_1 includes equivalent inertia of all elements that move uniformly with the motor disk; J_y and J_z are the pendulum moments of inertia, relative to its center of mass; l_{cg} is the center of gravity of the combined pendulum rod and weight ($m = m_r + m_w$); g is the gravity constant; and $u(t)$ is the control effort. For more details on parameters and modeling, see [18].

As equations (1) and (2) suggest, the state vector is defined by

$$\mathbf{x}^T(t) = [\theta_1(t) \ \dot{\theta}_1(t) \ \theta_2(t) \ \dot{\theta}_2(t)]. \quad (3)$$

Our control purpose is to navigate the load disk, induced by the second pendulum. This scenario is defined considering the load disk plus the main pendulum as the Segway, and the second pendulum as the person driving it. So, the feedback control must consider the position $\theta_2(t)$ and velocity $\dot{\theta}_2(t)$ of the main pendulum induced by the second pendulum (by hand in the experiment, but in the future modified using radio control). Also, the velocity of the load $\dot{\theta}_1(t)$ has to be taken into account, because the velocity of the Segway must be controlled.

A linear output feedback strategy is adopted to simplify the approach. In fact, it is convenient to point that firstly the H_∞ linear controller clammed in [12] was implemented in our Furuta experiment. Notwithstanding, the experimental test indicates that the closed-loop system was unstable. Also, when

the state-feedback control was implemented (full information case), the disk returned to its equilibrium position, and we did not obtain navigation (or Segway displacement). To obtain navigation, we have to ignore the load disk position $\theta_1(t)$. That is the reason to consider an output feedback control defined by $u(t) = \mathbf{K}\mathbf{y}(t)$, with $\mathbf{y}^T(t) = [\dot{\theta}_1 \ \theta_2 \ \dot{\theta}_2]$. In matrix notation, we have:

$$\mathbf{y}(t) = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{C}_2} \mathbf{x}(t) \quad (4)$$

Remark that \mathbf{C}_2 is right-hand invertible, that is, there exists \mathbf{C}_2^{-1} such that $\mathbf{C}_2 \cdot \mathbf{C}_2^{-1} = \mathbf{Id}_3$. This property will be used in the control synthesis.

Only the load disk angle position (θ_1) and the main pendulum angle position (θ_2) are available by the experimental platform, but velocities will be required in the control design. So, an observer is constructed to estimate the velocities $\dot{\theta}_1$ and $\dot{\theta}_2$ [19] .

3.2 Control objective

The control aim of this work is to design a robust control verifying two properties. One is to ensure local stability. The other is the requirement imposed to the control design to be robust in front of \mathcal{L}_2 disturbance. The problem of robust controller with guaranteed H_∞ performance is addressed to answer this question: Does there exist a feedback control such that the H_∞ norm of the closed-loop system from input disturbance named $w(t)$ to output $z(t)$ is less than some prescribed value γ ? [12]. In order to solve this problem, the LMI techniques are used.

Let us consider system (1)-(2). By model variables stated previously, the vector state $\mathbf{x}^T(t)$ and the output $\mathbf{y}(t)$ are defined in (3) and (4), respectively. Consider $\mathbf{z}^T(t) = [\theta_1(t) \ \theta_2(t) \ u(t)]$ the virtual output to be compared to the \mathcal{L}_2 perturbation $w(t)$ (induced by the second pendulum). Then, the state-space representation of system (1)-(4) yields

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}_2 u(t) + \mathbf{B}_1 w(t) , \\ \mathbf{z}(t) = \mathbf{C}_1 \mathbf{x}(t) + \mathbf{D}_{12} u(t) , \\ \mathbf{y}(t) = \mathbf{C}_2 \mathbf{x}(t) , \end{cases} \quad (5)$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\bar{J}_z & -m^2 l_{cg}^2 R_h g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & m R_h l_{cg} & m l_{cg} g (\bar{J}_1 + J_y) & 0 \end{bmatrix} ,$$

$$\mathbf{B}_1 = \mathbf{B}_2 = \begin{bmatrix} 0 \\ \bar{J}_z \\ 0 \\ -m R_h l_{cg} \end{bmatrix} ,$$

$$\mathbf{C}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } \mathbf{D}_{12} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} .$$

Our control objective is to design an output control matrix \mathbf{K} such that the

controller

$$u(t) = \mathbf{K}\mathbf{y}(t) \quad (6)$$

stabilizes the system (5) under \mathcal{L}_2 disturbances, employing H_∞ -LMI theory. A practical way to solve this problem is to consider a Lyapunov function $V(\mathbf{x}(t))$ such that for any nonzero $\mathbf{x}(t)$ and input $w(t) \in \mathcal{L}_2$, the following condition holds:

$$\frac{d}{dt}V(\mathbf{x}(t)) + \gamma^{-1} \mathbf{z}^T(t)\mathbf{z}(t) - \gamma w^T(t)w(t) < 0 . \quad (7)$$

Then, an H_∞ performance bound for the closed-loop system (5)–(6) is ensured (see [15] for details). If there exists a matrix \mathbf{K} such that (7) holds, then the control law $u(t) = \mathbf{K}\mathbf{y}(t)$ is said to be an H_∞ controller for the system (5), that is, the system is internally stable with H_∞ norm less than γ , i.e., $\|\mathbf{z}\|_\infty \leq \gamma^2 \|w\|_\infty$ for $w \in \mathcal{L}_2$.

From equations (5) and (6), we obtain the closed-loop system:

$$\begin{cases} \dot{\mathbf{x}}(t) = (\mathbf{A} + \mathbf{B}_2\mathbf{K}\mathbf{C}_2)\mathbf{x}(t) + \mathbf{B}_1w(t) , \\ \mathbf{z}(t) = \mathbf{C}_1\mathbf{x}(t) + \mathbf{D}_{12}u(t) . \end{cases}$$

Imposing the H_∞ condition (7), the term $\mathbf{B}_2\mathbf{K}\mathbf{C}_2$ difficulties the evaluation of the control gain matrix. In our case, we use that \mathbf{C}_2 is a right-hand invertible matrix to reduce the output feedback problem with not complete information, to a full information problem [12]:

$$\mathbf{u}(t) = \underbrace{\mathbf{K}\mathbf{C}_2}_{\bar{\mathbf{K}}} \mathbf{x}(t) \Rightarrow \mathbf{u}(t) = \bar{\mathbf{K}}\mathbf{x}(t) .$$

We solve then our problem for $\bar{\mathbf{K}}$ using the LMI theory, deriving an LMI procedure for the auxiliary gain matrix $\bar{\mathbf{K}}$.

Theorem 1. Consider the Furuta pendulum system (5)–(6). If there exist

$\gamma > 0$, matrices \mathbf{N} , $\mathbf{Y} > 0$ symmetric and \mathbf{V} regular such that the LMI

$$\begin{bmatrix} -(\mathbf{V}^T + \mathbf{V}) & * & * & * & * & * \\ \mathbf{AV} + \mathbf{Y} + \mathbf{B}_2\mathbf{N} & -\mathbf{Y} & * & * & * & * \\ 0 & \mathbf{B}_1^T & -\gamma & * & * & * \\ \mathbf{C}_1\mathbf{V} & 0 & 0 & -\gamma & * & * \\ \mathbf{N} & 0 & 0 & 0 & -\gamma & * \\ \mathbf{V} & 0 & 0 & 0 & 0 & -\mathbf{Y} \end{bmatrix} < 0, \quad (8)$$

is feasible, then the inequality (7) holds, with Lyapunov function defined as $V(\mathbf{x}) = \mathbf{x}^T \mathbf{P} \mathbf{x}$ with $\mathbf{P} := \mathbf{Y}^{-1}$, and $\bar{\mathbf{K}} := \mathbf{NV}^{-1}$. Consequently, $u(t)$ is an H_∞ controller defined by:

$$u(t) = \mathbf{NV}^{-1}\mathbf{C}_2^{-1} \mathbf{y}(t). \quad (9)$$

The proof of this result is straightforward from [6].

4 Experimental realization

To test the obtained theoretical results applied to the new laboratory device, the controller effectiveness is studied experimentally. We design the control (6) via the resolution of the LMI state in *Theorem 1*. Experiments have been performed on an ECP Model 220 industrial emulator with Furuta pendulum that includes a PC-based platform and DC brushless servo system [18].

The mechatronic system includes a motor used as servo actuator, a power amplifier and two encoders which provide accurate position measurements; i.e., 4000 lines per revolution with 4X hardware interpolation giving 16000counts per revolution to each encoder; 1 count (equivalent to 0.000392 radians or 0.0225 degrees) is the lowest angular measurable [18]. The second pendulum (external disturbance) includes an angular position sensor measured in radians. The drive

and load disks were connected via a 4 : 1 speed reduction (Figure 2).

In the experiments, the pendulum is set to the following parameters: $y_r = 42 \text{ cm}$, $y_m = 32 \text{ cm}$ (Figure 6). The parameters were taken from [18]:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1.1379 & -28.769 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.7219 & 50.229 & 0 \end{bmatrix},$$

$$\mathbf{B}_1 = \mathbf{B}_2 = \begin{bmatrix} 0 \\ 318.7 \\ 0 \\ -202.2 \end{bmatrix}.$$

Note that the perturbation and the control law enter to the system by the same channel ($B_1 = B_2$). Using the technique presented in *Theorem 1* and solving (8) with the LMI Matlab Toolbox [20], the next H_∞ controller was obtained: $\bar{\mathbf{K}} = [0.38 \quad 0.43 \quad 6.38 \quad 1.09]$. So, the output control (9) is defined as:

$$u(t) = [0.43 \quad 6.38 \quad 1.09] \mathbf{y}(t), \quad (10)$$

with H_∞ gain $\gamma = 8.2$. This expression may be used directly for control modeling, scaled by appropriate system gains (amplifier and software gains and motor torque constants). In the experiments, the controller in (10) was multiplied by 0.3 to compensate these gains.

Because velocity measurements are not available, the following controller realization is developed, where the velocity part is replaced by a first-order

linear compensator:

$$\begin{aligned} u &= 0.3 (0.43 \dot{x}_1 + 6.38 \theta_2 + 1.09 \dot{x}_2) , \\ \dot{x}_1 &= -10x_1 + 5\theta_1 , \\ \dot{x}_2 &= -10x_2 + 5\theta_2 . \end{aligned} \tag{11}$$

The terms x_1 and x_2 are auxiliary variables and their differential equations are solved numerically from θ_1 and θ_2 measurements. The above equations are a direct implementation of velocity observers given in [19], where the parameter -10 was set according to [21] and the gain 5 was adjusted experimentally¹. This approach goes along a two independent steps design procedure: a) design an output state-feedback controller \mathbf{K} , and b) construct a velocity observer. This design obeys the separation principle (see [21] and [22]).

In this experiment we try to simulate the Segway navigation, studying the load disk displacement when the main pendulum is strongly perturbed. Figure 7 presents a traveling from the video <http://youtu.be/SHQdW3k3qiE>, where the complete navigation experiment can be seen. Figures 8-10 show the load and principal pendulum displacement (as Segway navigation), and the control effort. Remark that at $t = 8$ sec, we return the second pendulum to its initial position (as Figure 7 shows). Then, the load disk remains in its final position (the Segway does not move if the person comes back to its upright position). This phenomena can be seen in Figure 8.

Figure 7: Clockwise direction. Pictures from <http://youtu.be/SHQdW3k3qiE>.

¹This consist to locate the observer poles from 5 to 10 times far away with respect to the vertical-axis-closest pole of $(A + B_2K)$.

Figure 8: Load disk position (simulation of the Segway navigation).

Figure 9: Main pendulum position (simulation of driving the Segway).

Figure 10: Control effort.

5 Conclusion

A new experimental set up for the Furuta pendulum was developed to validate controller performance under a kind of mass perturbation similar to the one used to navigate the Segway personal transportation unit. To simulate the Segway behavior, a second pendulum was coupled to the main one. The output control was designed in order to ensure stability and robustness. Experimental results demonstrate that the objectives have been achieved. Hence, the mechanical experiment provides interesting new results in the control automatic new developments.

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